

School of Mathematics

FACULTY OF MATHEMATICS AND PHYSICAL SCIENCES



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# The squizzel and its friends

John Paul Gosling (University of Leeds)

In 2004, an eminent scholar published an entertaining article in a special issue of a journal that was supposed to be about competing views on representing uncertainty.

How many legs does a squizzel have?

First tell me what a squizzel is.

Well, just use your own idea of what you think a squizzel is, and tell me how many legs it has.

.....

# Squizzels



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Squizzels according to Google.

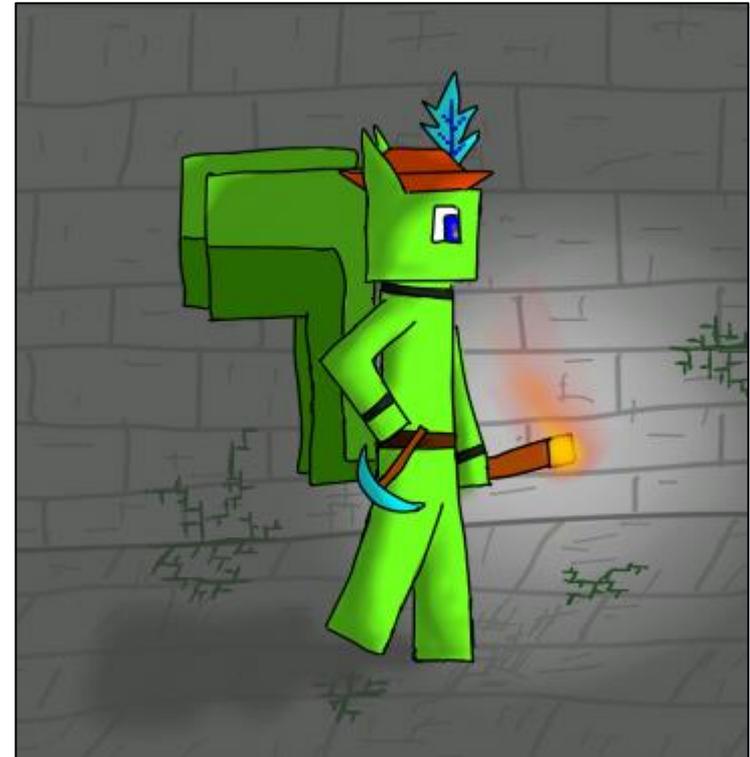


# Squizzels



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Squizzels according to Google.



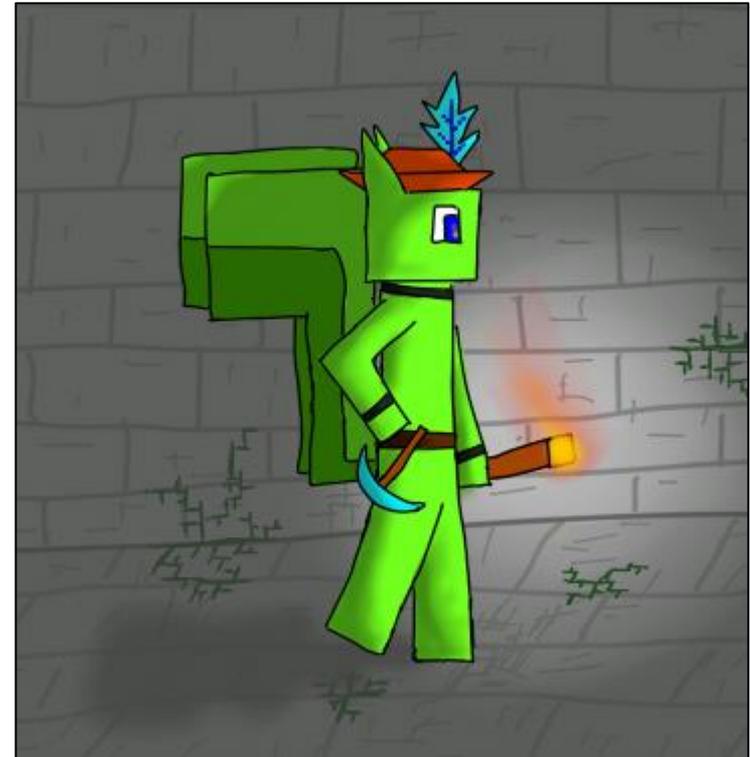
Squizzels don't exist, and their legs cannot be counted.

# Squizzels



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Squizzels according to Google.



Squizzels don't exist, and their legs cannot be counted.

Squizzels exist in our minds and are not readily comparable across different individuals.

Probability gives us a perfect way to quantify uncertainty because it is

- 1) measurable,
- 2) comparable across different individuals,
- 3) able to cover all types of uncertainty.

When combining probabilities over multiple experts, weights and scores are utilised that do not exhibit properties (1) and (2).

Let's restrict ourselves to the linear opinion pool of Stone (1961).

$$f_{pooled} = \sum_{experts} w_i f_i$$

We may believe that some experts have more to offer than others.

How can we determine the  $w_i$  in a consistent and defensible way?

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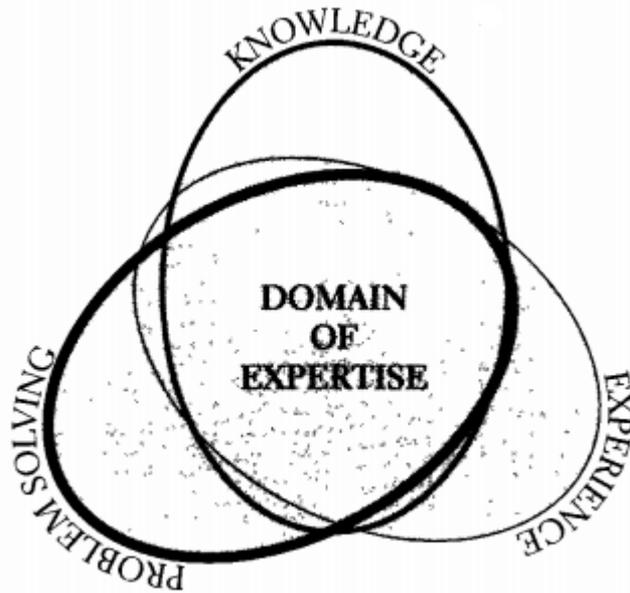
Is there an objective and unambiguous way of measuring this?

# Measures of expertise

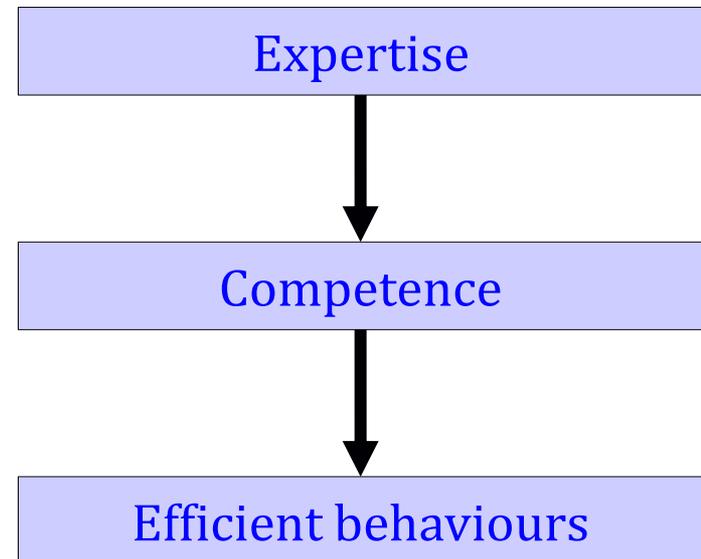


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Herling (2000) gives us:



“three foundational concepts of expertise”.



# Measures of expertise



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Germain and Tejada (2012) conclude that “an instrument measuring employee expertise, as perceived by another employee, can be developed”.

1. This person has knowledge that is specific to his or her field of work.
2. This person shows that they have the education necessary to be an expert in their field.
3. This person has knowledge about their field.
4. This person conducts research related to their field.
5. This person has the qualifications required to be an expert in their field.
6. This person has been trained in his or her area of expertise.
7. This person is ambitious about their work in the company.
8. This person can assess whether a work-related situation is important or not.
9. This person is capable of improving himself or herself.
10. This person is charismatic.
11. This person can deduce things from work-related situations easily.
12. This person is intuitive in their job.
13. This person is able to judge what things are important in their job.
14. This person has the drive to become what he or she is capable of becoming in their field.
15. This person is self-assured.
16. This person has self-confidence.
17. This person is an expert who is outgoing.
18. This person can talk his or her way through any work-related situation.

**Generalised expertise measure**

# Measures of expertise



Haakma (2011) proposed using some readily-measurable quantities to derive expert weights.

Table 3 Calibration factors

Years of experience (weight 0.45)		Average number of MRI's examined per week (weight 0.45)		Examining MRI's in other areas (weight 0.1)	
$X < 3$	1	$X < 5$	1	$X = 0$	1
$X \geq 3$	2	$5 \leq X < 10$	2	$X > 0$	2
		$10 \leq X$	3		

# Measures of expertise



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X=>3	2	5<=X<10	2	X>0	2
		10<=X	3		

Naseri and Barabady (2016) were far more direct in one of their proposals:

$$w_i = \frac{y_i}{\sum_j y_j}$$

where  $y_i$  is expert  $i$ 's number of years of experience.



# Measures of expertise

Burgman et al. (2011) looked at some measures of expertise including an eleven point expertise rating scale.

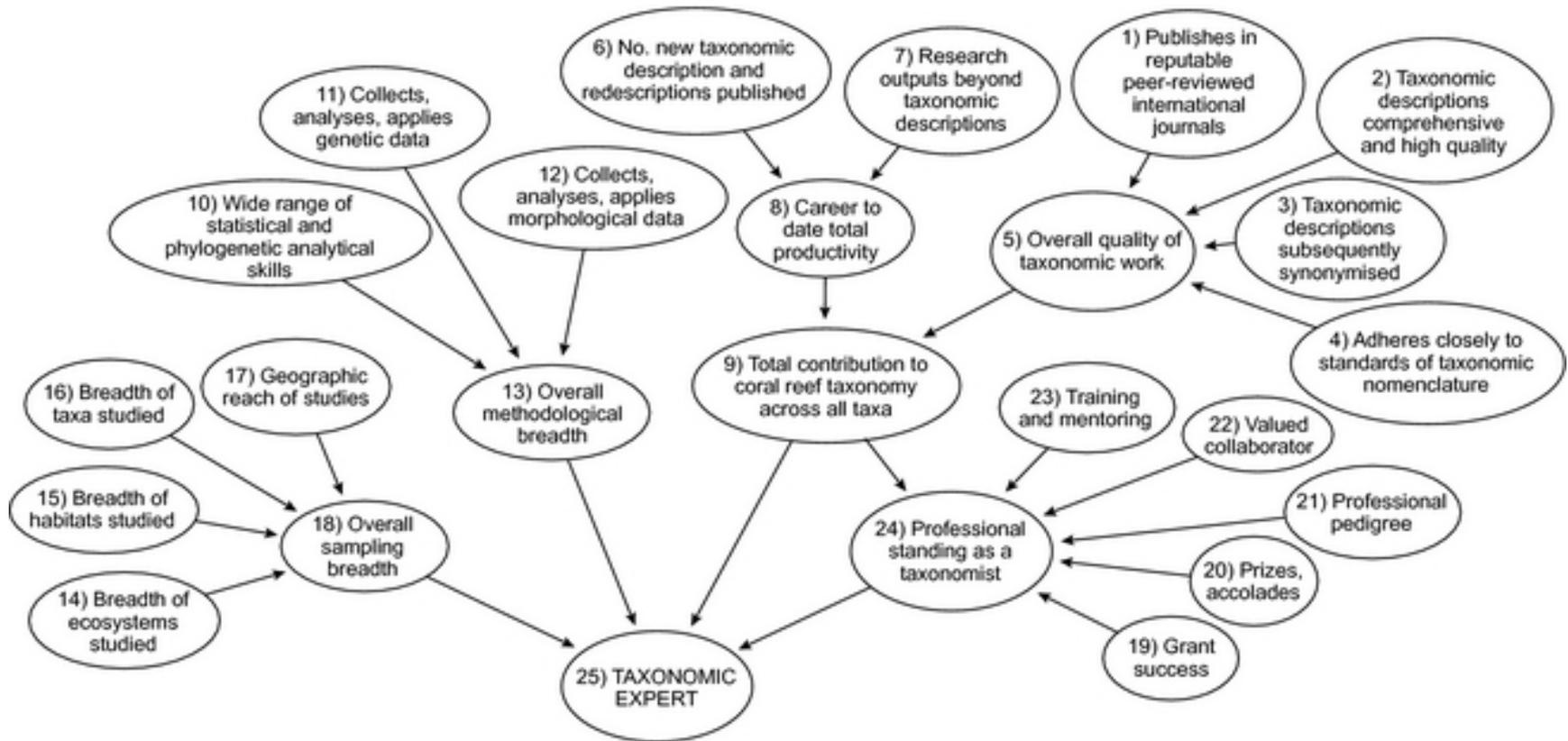
“Participants were then asked to privately rank themselves and the other participants on an 11-point scale (0= ‘no expertise’, 5= ‘moderate expertise’, 10= ‘highly expert’)”.

Workshop	Peer assessment versus years of experience	Peer assessment versus number of publications	Peer assessment versus qualifications	Self assessment versus peer assessment
1	<b>0.550 (n = 20)</b>	0.348 (n = 21)	<b>0.556 (n = 21)</b>	<b>0.675 (n = 21)</b>
2	<b>0.487 (n = 19)</b>	<b>0.587 (n = 21)</b>	0.064 (n = 20)	<b>0.684 (n = 24)</b>
3	<b>0.514 (n = 13)</b>	-0.123 (n = 13)	0.019 (n = 13)	<b>0.853 (n = 13)</b>
4	<b>0.591 (n = 25)</b>	<b>0.500 (n = 25)</b>	<b>0.489 (n = 25)</b>	<b>0.899 (n = 25)</b>
5	<b>0.836 (n = 20)</b>	0.309 (n = 17)	0.203 (n = 20)	<b>0.853 (n = 20)</b>
6	<b>0.620 (n = 14)</b>	0.289 (n = 14)	0.074 (n = 14)	<b>0.944 (n = 14)</b>



# Measures of expertise

Caley et al. (2014) used a Bayesian network to attempt to define expertise in a particular field.



# Measures of expertise



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Winkler (1968) talked about the concepts of a “better assessor” and the “goodness” of an expert, and he considered various options for weighting:

Equal,

Proportional to external ranking,

Proportional to a self-rating,

Derived from a measures of predictive performance.

**But based ultimately on the judgement user’s opinions.**

# Confidence



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What is my probability of some event occurring?

$p_1$

# Confidence



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What is my probability of some event occurring?

$p_1$

What is my confidence in that judgement?

$c_1$

# Confidence



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What is my probability of some event occurring?

$p_1$

What is my confidence in that judgement?

$c_1$

What is my confidence in another expert's judgement ( $p_2$ )?

$c_2$

What is my probability of some event occurring?

$p_1$

What is my confidence in that judgement?

$c_1$

What is my confidence in another expert's judgement ( $p_2$ )?

$c_2$

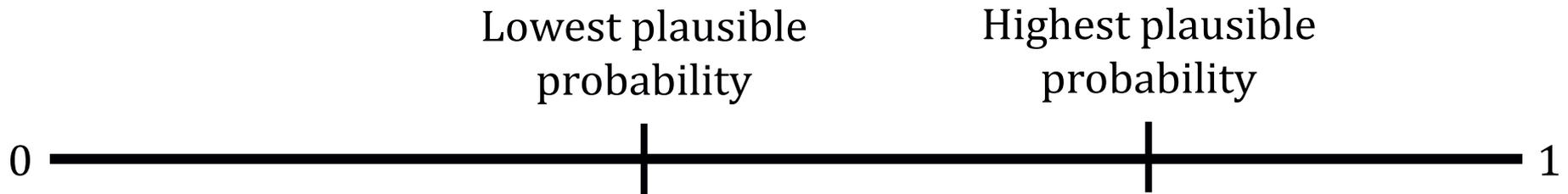
What is meant by “confidence”?

# Capturing confidence



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Consider a binary event.

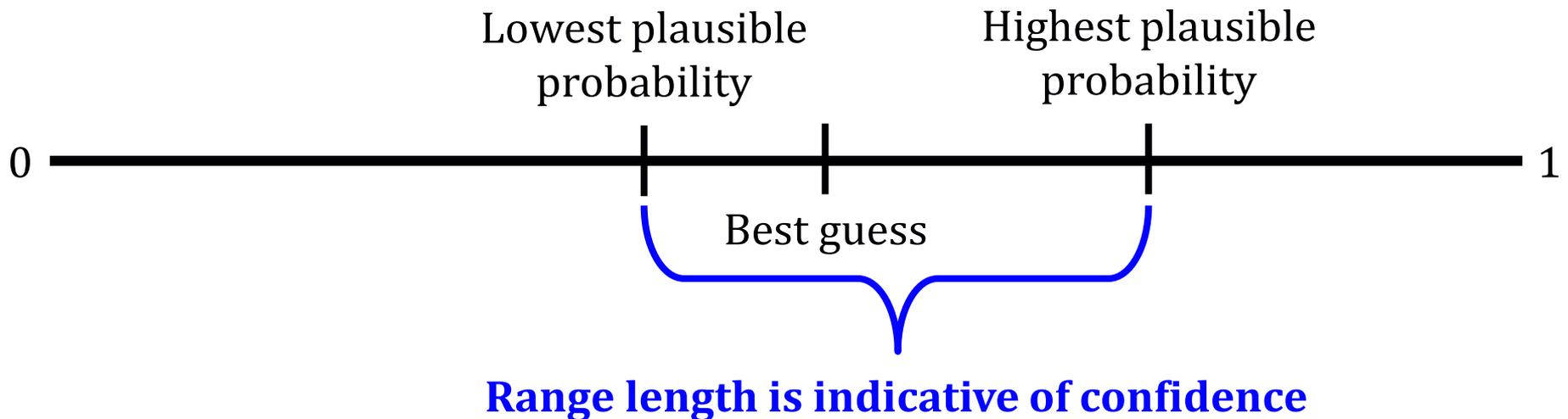


# Capturing confidence



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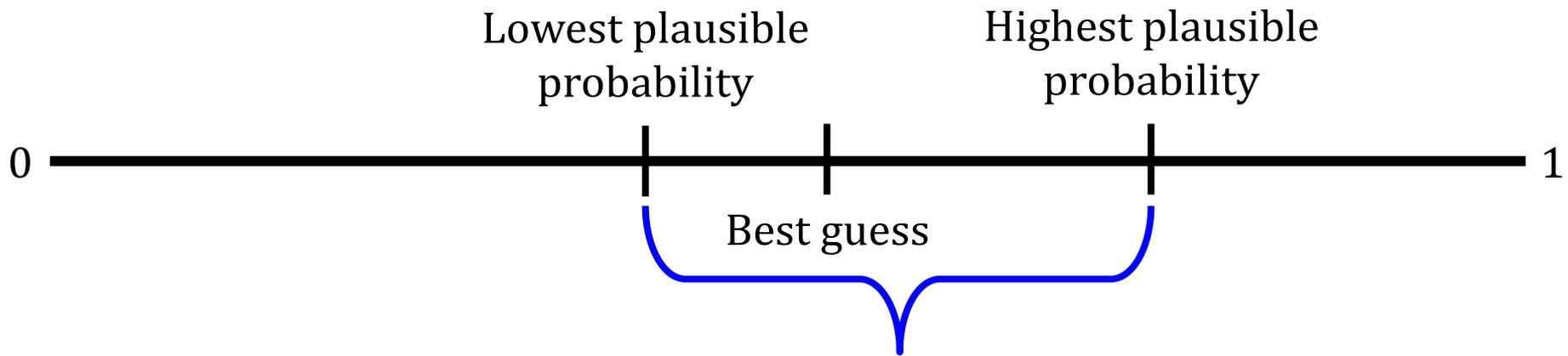


# Capturing confidence



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Consider a binary event.



**Range length is indicative of confidence**

What is meant by “plausible”?

And should we be using imprecise probability?

# Relative importance



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DeGroot (1974) introduces a model for reaching a consensus distribution that has been revisited several times in the past 40 years.

Rounds of consensus forming are modelled by

$$\mathbf{F}^{(n)} = \mathbf{P}\mathbf{F}^{(n-1)} = \mathbf{P}^n \mathbf{F}$$

where  $\mathbf{F}$  is a matrix encapsulating the experts' judgements about some quantity of interest and  $\mathbf{P}$  is a matrix including weights given by the experts on the basis of **relative importance** of the opinions of each expert.

# Manipulating weights



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We should worry about conflicts of interest in expert elicitation exercises.

An unscrupulous expert could adjust their weights or ratings of confidence to unduly influence the resulting distribution.

**Extreme case:**

$$P = \begin{pmatrix} 0.2 & 0.4 & 0.4 \\ & & \end{pmatrix}$$

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$$\mathbf{P} = \begin{pmatrix} 0.2 & 0.4 & 0.4 \\ 0.2 & 0.7 & 0.1 \end{pmatrix}$$

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# Manipulating weights



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An unscrupulous expert could adjust their weights or ratings of confidence to unduly influence the resulting distribution.

**Extreme case:**

$$\mathbf{P}^n \rightarrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

# Bayesian approaches

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Moving away from linear pooling, we could model the experts' judgements as data, but the problem of relative importance does not disappear.

# Bayesian approaches



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Moving away from linear pooling, we could model the experts' judgements as data, but the problem of relative importance does not disappear.

Albert et al. (2012) ask each expert for a number in  $(0,1)$  that quantifies the confidence in their response.

Gelfand et al. (1995) says that we need assess the extent of knowledge about the unknown as well as sources of information and experience that have formed this knowledge.

Lindley (1983) suggests that correlation across different experts' judgements will be difficult to assess and that an inverse Wishart distribution may be inappropriate to model our uncertainty about this.

# Other approaches



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There have been many other attempts to rate experts based on coherence, calibration, counteracting conflicts of interest and other scoring rules...

Clemen and Winkler (1990) consider the principle of unanimity as a guide to weighting experts.

Wright et al. (1994) devised a scoring rule based upon expert coherence when assessing probabilities in different ways.

Boutilier (2012) looks at devising scoring rules that actively seek to counter experts' conflicts through considering utilities.

Fisher et al. (2012) ask for an estimate of how sure they are that the real value lies between these bounds; here this is termed sureness and is expressed as a percentage.

# Summary



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If I was a decision maker, I would be worried about:

- 1) Ambiguities in questions posed to experts,
- 2) Ambiguities in interpreting experts' answers,
- 3) Potential for the manipulation of results.

These concerns can be avoided by using procedures that **reward honesty** and **eliminate ambiguities** (and favour people that can make probabilistic judgements).

# References



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## Plus some that I missed off earlier slides:

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